

MHD Free Convective Radiative and Chemicaly Reactive Flow Over a Vertical Porous Surface in the Presence of Diffusion-Thermo Effect

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Abstract—The main purpose of this work is to investigate the porous medium and diffusion thermo effects on unsteady combined convection magneto hydrodynamics boundary layer flow of viscous electrically conducting fluid over a vertical porous surface in the presence of first order chemical reaction and thermal radiation. The slip boundary condition is applied at the porous surface. A uniform Magnetic field is applied normal to the direction of the fluid flow. The non-linear coupled partial differential equation are solved by perturbation method and obtained the expressions for concentration, temperature and velocity fields. The rate of mass transfer in terms of Sherwood number h , the rate of heat transfer in terms of Nusselt number Nu and the Skin friction coefficient C_f are also derived. The Profiles of fluid flow and derived quantities for various values of physical parameters are presented and analyzed.

Keywords—Diffusion-thermo effect, Thermal radiation, Chemical reaction, Magnetic field, Absorption of Radiation.

Nomenclature

A	Suction velocity parameter
B_0	Magnetic field of uniform strength
C	Species concentration
C^*	Dimensional concentration
C_p	Specific heat at constant pressure
C_s	Concentration susceptibility
C_w	Species concentration at plate
C_∞	Species concentration far away from the plate
C_f	Skin friction coefficient
D	Molecular diffusivity
$e_{b\lambda}$	Planck's function
g	Acceleration due to gravity

Gm	Solutal Grashof number
Gr	Thermal Grashoff number
D_m	Coefficient of mass diffusivity
K_T	Thermal diffusion ratio
$K_{\lambda w}$	Absorption coefficient at the wall
M	Magnetic field parameter
F	Radiation parameter
Nu	Nusselt number
n^*	Constant
P^*	Pressure
Pr	Prandtl number
Q	Heat source/sink parameter
Q^*	Sink strength
Q_0	Dimensional heat absorption coefficient
Q_1	Absorption of radiation parameter
Q_1^*	Coefficient of proportionality for the absorption
q_r^*	Radiative heat flux
Sc	Schmidt number
Sh	Sherwood number
T	Fluid Temperature
T^*	Temperature of the fluid near the plate
T_w	Temperature at the wall
T_∞	Temperature far away from the plate
u^*, v^*	Components of dimensional velocities
u^*	Velocity of the fluid along x^*
U_∞	Free stream dimensional velocity
U_p	Wall dimensional velocity
v^*	Velocity of the fluid along y^*
U_0	Scale of free stream velocity
V_0	Section velocity
C_p	Specific heat at a constant pressure
C_v	Specific heat at a constant volume
x^*, y^*	Dimensional the distances along and perpendicular to the plate

Re_x Local Reynolds number

Greek Symbols

μ Fluid dynamic viscosity

α Fluid thermal diffusivity

ν Coefficient of kinematic viscosity

β Coefficient of Volumetric expansion for the heat transfer

β^* Coefficient of Volumetric expansion for the fluid

β_0 Magnetic field coefficient

β_T, β_c Thermal and concentration expansion coefficients

ρ Density of the fluid

σ Electrically conductivity of the fluid

η Dimensionless normal distance

Fluid currents formed in a fluid-saturated porous medium during convective heat transfer have many important applications, such as oil and gas production, central grain storage, porous insulation, and geothermal energy. The study of natural convection through porous medium also throws some light on the influence of environment such as temperature and pressure on the germination of seeds. In many situations, the heat and mass transfer on the hydromagnetic flow near the vertical plate is encountered e.g., the cooling of nuclear reactor with electrically conducting coolants such as liquid sodium and mercury, studied by Rath and Parida [1]. Raptis [2], Jha and Prasad [3] have studied the steady free-convection flow and mass transfer through a porous medium bounded by an infinite vertical plate for the flow near plate by using the model of Yamamoto and Iwamura [4]. Gebhart and Pera [5] have studied the laminar flows which arise in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species.

As stated by Pal and Talukdar [6], convection in porous media has gained significant attention in recent years because of its importance in engineering applications such as solid matrix heat exchangers, geothermal systems, thermal insulations, oil extraction and store of nuclear waste materials. Convection in porous media also is applied to underground coal gasification, ground water hydrology, iron blast furnaces, wall cooled catalytic reactors, solar power collectors, energy efficient drying processes, cooling of electronic equipment's and natural convection in earth's crust. Reviews of the applications associated to convective flows in porous media can be found in Nield and Bejan [7]. The fundamental problems of flow through and past porous media has been studied extensively over the years both theoretically and experimentally by Cheng [8] and Rudraiah [9]. The effect of Beavers-Joseph slip velocity and transverse magnetic

\emptyset Heat source parameter

\emptyset_1 Slip parameter

κ Thermal conductivity

Superscripts

' Differentiation with respect to y

*

Subscripts

p Plate

w Wall condition

∞ Free stream condition

I. INTRODUCTION

field on an electrically conducting viscous fluid in a horizontal channel bounded on both sides porous substrates of finite thickness was studied Rudraiah et al. [10], which is equivalent to the problem of forced convection where the momentum equation is independent of concentration distribution and the diffusion equation is coupled with the velocity distribution using Beavers-Joseph slip condition at the porous interface.

In many chemical engineering processes, the chemical reaction occurs between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications e.g., manufacturing of ceramics or glassware, polymer production, and food processing. Chemical reactions can be codified as either homogeneous or heterogeneous processes. This depends on whether these occur at an interface or as a single phase volume reaction. Many transport processes exist in a nature and mass transfer as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Apelblat [11] studied analytical solution for mass transfer with a chemical reaction of the first order. Das et al. [12] have analyzed the effects of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Chambre and Young [13] have analyzed a first order chemical reaction in the neighborhood of a horizontal plate.

Radiative convective flows are encountered in various ways in the environment e.g., heating and cooling chambers, evaporation from large open water reservoirs, fossil fuel combustion energy processes, solar power technology, astrophysical flows, and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines, and various propulsion devices for satellites, missiles, aircraft and various space vehicles are examples of such

engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

The thermal radiation effects of an optically thin gray gas bounded by a stationary vertical plate was studied by England and Emery [14]. The radiative natural convective flow of an optically thin gray-gas past a semi-infinite vertical plate was considered by Soundalgekar and Takhar [15]. In all above studies, the stationary vertical plate is considered. The effects of the thermal radiation and free convection flow past a moving vertical plate studied by Raptis and Perdakis [16]. Ibrahim et al. [18] have recently reported computational solutions for transient reactive magneto hydrodynamic heat transfer with heat source and wall flux effects. They have also analyzed the effects of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Due to the importance of Soret (thermal-diffusion) and Dufour (diffusion thermo) effects for the fluids with very light molecular weight as well as medium molecular weight many investigators have studied and reported results for these flows of whom the names are Dursunkaya and Worek [19], Anghel *et al.* [20], Postelnicu [21] are worth mentioning.

Chamkha[24] investigated unsteady convective heat and mass transfer past a semi-infinite porous moving plate with heat absorption. Chamkha[25] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. Mohamed [26] has discussed double diffusive convection radiation interaction on unsteady MHD flow over a vertical moving porous plate with heat generation and Soret effects. Muthucumaraswamy and Janakiraman[27] studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion. Rajesh and Varma[28] studied thermal diffusion and radiation effects on MHD flow past a vertical plate with variable temperature. Kumar and Varma[29] investigated thermal radiation and mass transfer effects on MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature and mass diffusion. Raptis et al.[30] studied the hydromagnetic free convection flow of an optically thin gray gas taking into account the induced magnetic field in the presence of radiation and the analytical solutions were obtained by perturbation technique. Orhan and Ahmad [31] examined MHD mixed convective heat

transfer along a permeable vertical infinite plate in the presence of radiation and solutions are derived using Kellar box scheme and accurate finite-difference scheme. Ahmed [32] investigated the study of influence of thermal radiation and magnetic Prandtl number on the steady MHD heat and mass transfer mixed convection flow of a viscous, incompressible, electrically-conducting Newtonian fluid over a vertical porous plate with induced magnetic field.

When heat and mass transfer occur simultaneously in moving fluid, the relations between the fluxes and the driving potentials are of a more intricate nature. It has been observed that an energy flux can be generated not only by temperature gradients but also by concentration gradients. The energy flux caused by a concentration gradient is termed the diffusion-thermo (Dufour) effect. In most of the studies related to heat and mass transfer process, Soret and Dufour effects are neglected on the basis that they are of a smaller order of magnitude than the effects described by Fourier's and Fick's laws. But these effects are considered as second order phenomena and may become significant in areas such as hydrology, petrology, geosciences, etc. For fluids with medium molecular weight (H_2 , air), Dufour and Soret effects should not be neglected as indicated by Eckert and Drake [17]. Prakash et al. [33] analyzed Diffusion-Thermo and Radiation Effects on Unsteady MHD Flow through Porous Medium Past an Impulsively Started Infinite Vertical Plate with Variable Temperature and Mass Diffusion. To our best knowledge, the interaction between the diffusion-thermo and chemical reaction in the presence of porous medium, heat absorption/generation, thermal radiation, radiation absorption effects has received little attention. Hence, an attempt is made to study the diffusion-thermo effects on an unsteady MHD free convective heat and mass transfer flow of a viscous incompressible electrically conducting fluid through porous medium from a vertical porous plate with varying suction velocity in slip flow regime.

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II. MATHEMATICAL FORMULATION

Consider an unsteady two dimensional flow of an incompressible viscous, electrically conducting and heat-absorbing fluid past a semi-infinite vertical permeable plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of thermal and concentration buoyancy effects. The applied magnetic field is also taken as being weak so that Hall and ion slip effects may be neglected. We assume that the Dufour effects may be described by a second-order concentration derivative with respect to the transverse coordinate in the energy equation. Further to

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x^*} + v \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* + g\beta_T(T^* - T_\infty) + g\beta_c(C^* - C_\infty) \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{K}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q_r^*}{\partial y^*} - \frac{Q_0}{\rho C_p} (T^* - T_\infty) + \frac{Q_1^*}{\rho C_p} (C^* - C_\infty) + \frac{D_m}{C_s} \frac{K_T}{\rho C_p} \frac{\partial^2 C^*}{\partial y^{*2}} \quad (3)$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - R(C^* - C_\infty) \quad (4)$$

where x^* , y^* and t^* are the dimensional distances along x^* and y^* directions and dimensional time respectively, u^* and v^* are the components of dimensional velocities along x^* and y^* directions respectively, T^* is the dimensional temperature, C^* is the dimensional concentration, T_w and C_w are the temperature and concentration at the wall, C_∞ and T_∞ are the free stream dimensional concentration and temperature, ρ is the density, ν is kinematic viscosity, C_p is the specific heat at constant pressure, σ is the fluid electrical conductivity, B_0 is the magnetic induction, K^* is the permeability of the porous medium, q_r^* is radiative heat flux, Q_0 is the dimensional heat absorption coefficient, Q_1^* is the coefficient of proportionality for the absorption, R is the chemical reaction, β_T and β_c are the thermal and concentration expansion coefficients, D is the molecular diffusivity, D_m is the coefficient of mass diffusivity, K_T is the thermal diffusion ratio,

$Q_0(T^* - T_\infty)$ is assumed to be the amount of heat generated or absorbed per unit volume and Q_0 is a constant.

The radiative heat flux is considered, which is given by Cogley et al. [22], Pal and Talukdar [6] as

$$\frac{\partial q_r^*}{\partial y^*} = 4(T^* - T_\infty)I' \quad (5)$$

where $I' = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T^*} d\lambda$, $K_{\lambda w}$ the coefficient of absorption near the wall and $e_{b\lambda}$ is Planck's function.

Under the above stated assumption, the initial and boundary conditions for the velocity distribution involving slip flow, temperature and concentration distributions are defined as:

$$u^* = u_{slip}^* = \frac{\sqrt{k}}{\alpha} \frac{\partial u^*}{\partial y^*}, T^* = T_w + \epsilon(T_w - T_\infty)e^{n^*t^*}, \text{ at } y^* = 0 \quad (6)$$

$$C^* = C_w + \epsilon(C_w - C_\infty)e^{n^*t^*}, \text{ at } y^* = 0 \quad (7)$$

$$u^* = U_\infty^* = U_0(1 + \epsilon e^{n^*t^*}), T^* \rightarrow T_\infty,$$

$$C^* \rightarrow C_\infty, \text{ as } y^* \rightarrow \infty \quad (8)$$

From Eq. (1) it is clear that the suction velocity at the plate surface is either constant or a function of time only. Hence, it is assumed that

$$\vartheta^* = -V_0(1 + \epsilon A e^{n^*t^*}) \quad (9)$$

where V_0 is the mean suction velocity and $\epsilon A \ll 1$. The negative sign indicated that the suction velocity is directed towards the plate.

In the free stream Eq.(2) gives

$$-\frac{1}{\rho} \frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} + \frac{\sigma}{\rho} B_0^2 U_\infty^* \quad (10)$$

On introducing the non-dimensional quantities

$$u = \frac{u^*}{U_0}, \vartheta = \frac{\vartheta^*}{V_0}, y = \frac{v_0 y^*}{\nu}, U_\infty = \frac{U_\infty^*}{U_0},$$

our assumption that there is no applied voltage which implies the absence of an electric field. The plate is maintained at constant temperature T_w and concentration C_w , higher than the ambient temperature T_∞ and concentration C_∞ respectively. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant. Due to the semi-infinite plane surface assumption, the flow variables are functions of y^* and time t^* only. Under the usual boundary layer approximations the governing equations are governed by the following equations.

$$t = \frac{V_0^2 t^*}{v}, \theta = \frac{T^* - T_\infty}{T_w - T_\infty}, C = \frac{C^* - C_\infty}{C_w - C_\infty},$$

$$n = \frac{n^* v}{V_0^2}, Gr = \frac{\rho g v (T_w - T_\infty) \beta_T}{U_0 V_0^2},$$

$$Gm = \frac{\rho g v (C_w - C_\infty) \beta_C}{U_0 V_0^2}, M = \frac{\sigma v B_0^2}{\rho V_0^2},$$

$$Pr = \frac{v C_p}{K}, \phi = \frac{Q_0 v}{\rho V_0^2 C_p}, Q_1 = \frac{Q_1^* v (C_w - C_\infty)}{V_0^2 (T_w - T_\infty)},$$

$$F = \frac{4v I'}{V_0^2 \rho C_p}, Sc = \frac{v}{D}, \gamma = \frac{Rv}{V_0^2},$$

$$Du = \frac{D_m k_T (C_w - C_\infty)}{C_s K (T_w - T_\infty)} \quad (11)$$

In view of the above dimensionless variables, the basic field of Eqs. (2) to (4) can be expressed in dimensionless form as

$$\frac{\partial u}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + M(U_\infty - U) + \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC, \quad (12)$$

$$\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - F\theta + Q_1 C - \phi\theta + \frac{Du}{Pr} \left(\frac{\partial^2 C}{\partial y^2} \right) \quad (13)$$

$$\frac{\partial C}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} - \gamma C \quad (14)$$

The corresponding initial and boundary conditions in equations (6)-(8) in non-dimensional form are given below

$$u = u_{slip} = \phi_1 \frac{\partial u}{\partial y}, \theta = 1 + \epsilon e^{nt},$$

$$C = 1 + \epsilon e^{nt} \text{ at } y = 0 \quad (15)$$

$$U \rightarrow U_\infty = (1 + \epsilon e^{nt}), \theta \rightarrow 0, C \rightarrow 0$$

$$\text{as } y \rightarrow \infty \quad (16)$$

III. METHOD OF SOLUTION

The equations (12) to (14) are coupled non-linear partial differential equations whose solutions in closed-form are difficult to obtain. To solve these coupled non-linear partial differential equations, we assume that the unsteady flow is superimposed on the mean steady flow, so that in the neighbourhood of the plate, we have

$$u = f_0(y) + \epsilon e^{nt} f_1(y) + O(\epsilon^2) \quad (17)$$

$$\theta = g_0(y) + \epsilon e^{nt} g_1(y) + O(\epsilon^2) \quad (18)$$

$$C = h_0(y) + \epsilon e^{nt} h_1(y) + O(\epsilon^2) \quad (19)$$

By substituting the set of Eqs. (17)-(19) into Eqs.(12)-(14) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms in ϵ , we obtain

$$f_0'' + f_0' - M_1 f_0 = -M_1 - Gr g_0 - Gm h_0 \quad (20)$$

$$f_1'' + f_1' - M f_1 = -(M + n) - A f_0' - Gr g_1 - Gm h_1 \quad (21)$$

$$g_0'' + Pr g_0' - Pr(F + \phi) g_0 = -Pr Q_1 h_0 - Du h_0' \quad (22)$$

$$g_1'' + Pr g_1' - Pr(F + \phi + n) g_1 = -Pr Q_1 h_1 - Du h_1' - Pr A g_0' \quad (23)$$

$$h_0'' + Sch_0' - Sc \gamma h_0 = 0 \quad (24)$$

$$h_1'' + Sch_1' - Sc(\gamma + n) h_1 = -ASch_0' \quad (25)$$

where the prime denotes the differentiation with respect to y . Now the corresponding boundary conditions are

$$f_0 = \phi_1 f_0', f_1 = \phi_1 f_1', g_0 = 1,$$

$$g_1 = 1, h_0 = 1, h_1 = 1 \quad \text{at } y = 0 \quad (26)$$

$$f_0 = 1, f_1 = 1, g_0 \rightarrow 0, g_1 \rightarrow 0, h_0 \rightarrow 0,$$

$$h_1 \rightarrow 0, \text{ as } y \rightarrow \infty \quad (27)$$

By solving the set of Eqs.(20)- (25) with the help of boundary conditions (26)-(27), the following set of solutions are obtained

$$f_0 = 1 + B_4 e^{-m_6 y} - A_3 e^{-m_2 y} - A_2 e^{-P_1 y} \quad (28)$$

$$f_1 = 1 + B_5 e^{-m_3 y} + A_4 e^{-m_6 y} - A_{10} e^{-m_2 y} - B_1 e^{-P_1 y} - B_2 e^{-m_5 y} - B_3 e^{-m_4 y} \quad (29)$$

$$g_0 = (1 - A_1) e^{-m_2 y} + A_1 e^{-P_1 y} \quad (30)$$

$$g_1 = A_9 e^{-m_5 y} + A_6 e^{-P_1 y} + A_7 e^{-m_2 y} + A_8 e^{-m_4 y} \quad (31)$$

$$h_0 = e^{-P_1 y} \quad (32)$$

$$h_1 = (1 - A_5) e^{-m_4 y} + A_5 e^{-P_1 y} \quad (33)$$

$$u(y, t) = 1 + B_4 e^{-m_6 y} - A_3 e^{-m_2 y} - A_2 e^{-P_1 y} + e^{nt} (1 + B_5 e^{-m_3 y} + A_4 e^{-m_6 y} - A_{10} e^{-m_2 y} - B_1 e^{-P_1 y} - B_2 e^{-m_5 y} - B_3 e^{-m_4 y}) \quad (34)$$

$$\theta(y, t) = (1 - A_1) e^{-m_2 y} + A_1 e^{-P_1 y} + e^{nt} (A_9 e^{-m_5 y} + A_6 e^{-P_1 y} + A_7 e^{-m_2 y} + A_8 e^{-m_4 y}) \quad (35)$$

$$C(y, t) = e^{-P_1 y} + e^{nt} ((1 - A_5) e^{-m_4 y} + A_5 e^{-P_1 y}) \quad (36)$$

The coefficient of Skin-friction, the rate of heat transfer in terms of Nusselt number and the rate of mass transfer are the important physical parameters for this kind of boundary layer flow. Hence these quantities are defined and derived as follows:

$$C_{fx} = \frac{\tau_w}{\rho u_0 v_0} = \left(\frac{\partial u}{\partial y} \right)_{at y=0} = -B_4 m_6 + A_3 m_2 + A_2 P_1 + e^{nt} (-B_5 m_3 - A_4 m_6 + A_{10} m_2 + B_1 P_1 + B_2 m_5 + B_3 m_4) \quad (37)$$

$$Nu_x = x \frac{\left(\frac{\partial T}{\partial y^*} \right)_{at y=0}}{(T_w - T_\infty)} \Rightarrow Nu_x / Re_x = \left(\frac{\partial \theta}{\partial y} \right)_{at y=0} = (-m_2 (1 - A_1) - A_1 P_1) + e^{nt} (-m_5 A_9 - P_1 A_6 - m_2 A_7 - m_4 A_8) \quad (38)$$

$$Sh_x = x \frac{\left(\frac{\partial C}{\partial y^*} \right)_{at y=0}}{(C_w - C_\infty)},$$

$$Sh_x / Re_x = \left(\frac{\partial C}{\partial y} \right)_{at y=0}$$

$$= -P_1 + e^{nt} (-m_4 (1 - A_5) - A_5 P_1) \quad (39)$$

Where $Re_x = \frac{V_0 x}{\nu}$ is the local Reynolds number.

IV. RESULTS AND DISCUSSION

The analytical solutions are performed for concentration, temperature and velocity for various values of fluid flow parameters such as Schmidt number Sc , chemical reaction parameter γ , Dufour number Du , Magnetic field parameter M , Heat absorption parameter ϕ , Radiation

absorption parameter Q_1 , Radiation parameter F , Porous permeability parameter ϕ_1 , Solutal Grashof number Gr , mass Grashof number Gm are presented in figures 1-15. Throughout the calculations the parametric values are chosen as $Pr = 0.71$, $A = 0.5$, $\varepsilon = 0.02$, $n = 0.1$, $t = 1$.

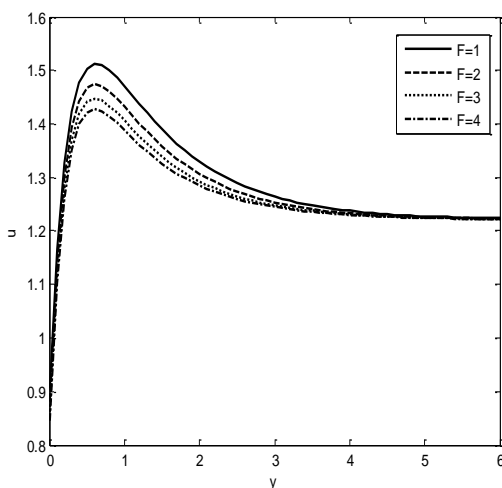


Fig.1: Velocity Profiles for various values of F

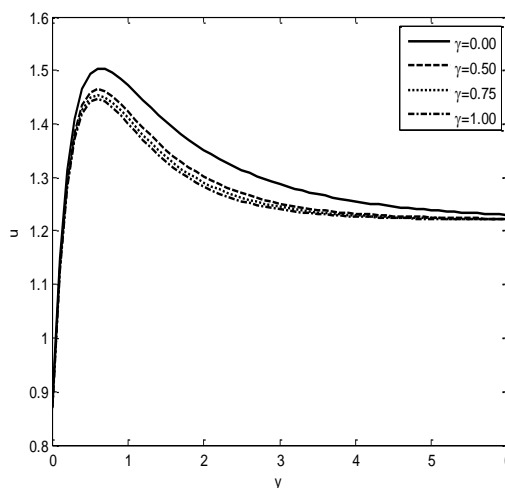


Fig.2: $Sc = 0.6$, $Gr = 4.0$, $Gm = 2.0$, $Q_1 = 2.0$, $\phi =$

with, $Gr = 4.0, Sc = 0.6, Gm = 2.0, Q_1 = 2.0, \phi = 1, M = 2.0, Du = 0.5, \phi_1 = 0.3, \gamma = 0.5$.

$1, M = 2.0, Du = 0.3, \phi_1 = 0.3, F = 2.0$.

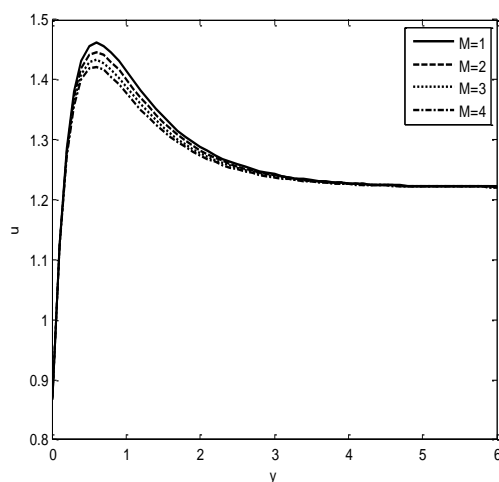


Fig.3: .Velocity profiles for various values of M with, $Sc = 0.6, Gr = 4.0, Gm = 2.0, Q_1 = 2.0, \phi = 1.0, Du = 0.3, \phi_1 = 0.3, F = 2.0, \gamma = 1$.

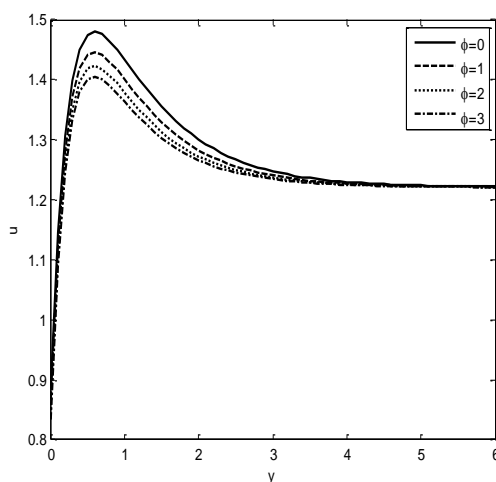


Fig.4: Velocity profiles for various values of ϕ with $Sc = 0.6, Gr = 4.0, Gm = 2.0, Q_1 = 2.0, Du = 0.3, \phi_1 = 0.3, F = 2.0, \gamma = 1, M = 2$.

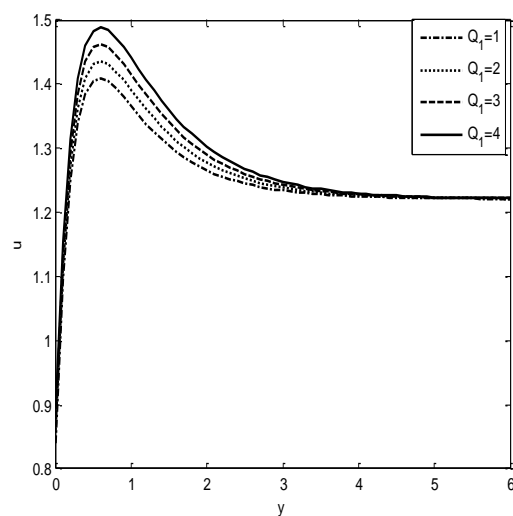


Fig.5: Velocity profiles for various values of Q_1 with $Sc = 0.6, Gr = 4.0, Gm = 2.0, M = 2.0, Du = 0.1, \phi_1 = 0.3, F = 2.0, \gamma = 1, \phi = 1.0$.

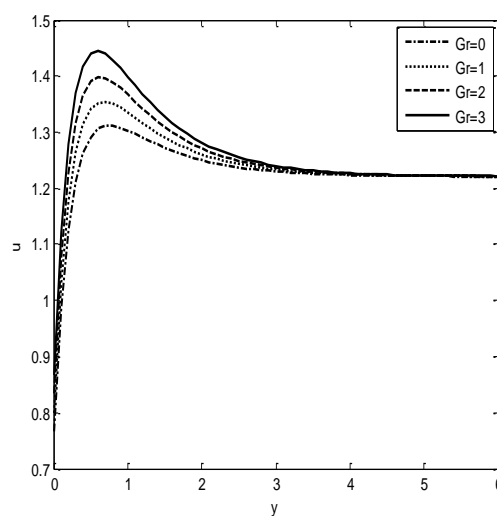


Fig.6: Velocity profiles for various values of Gr with $Sc = 0.6, Gm = 3.0, M = 2.0, Q_1 = 2.0, Du = 0.3, \phi_1 = 0.3, F = 2.0, \gamma = 1, \phi = 1.0$

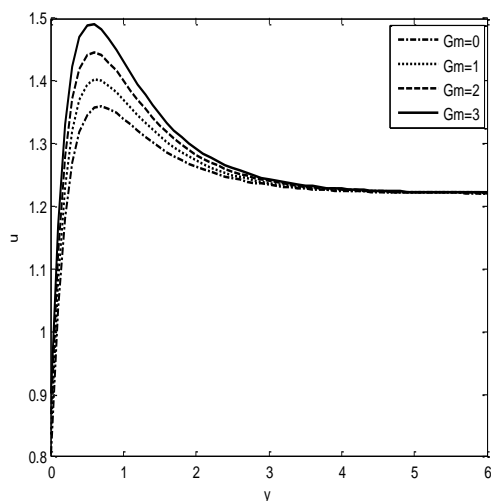


Fig.7: Velocity profiles for various values of Gm with $Sc = 0.6$, $Gr = 4.0$, $M = 2.0$, $Q_1 = 2.0$, $Du = 0.3$, $\phi_1 = 0.3$, $F = 2.0$, $\gamma = 1$, $\phi = 1.0$.

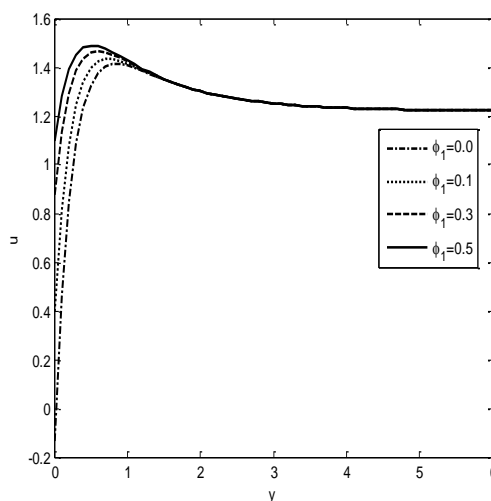


Fig.8: Velocity profiles for various values of ϕ_1 with $Sc = 0.6$, $Gr = 4.0$, $Gm = 3.0$, $M = 2.0$, $Q_1 = 2.0$, $Du = 0.3$, $F = 2.0$, $\gamma = 0.5$, $\phi = 1.0$.

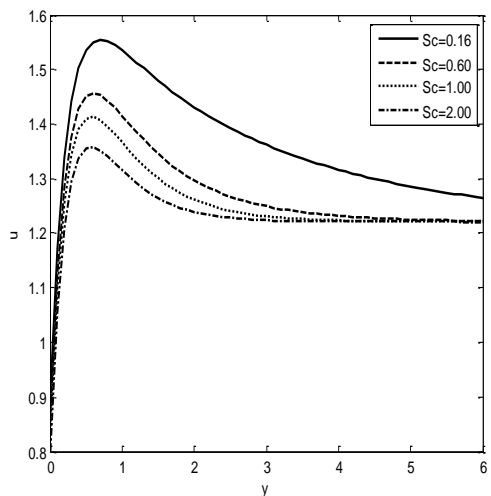


Fig.9: Velocity profiles for various values of Sc with $Gr = 2.0$, $Gm = 2.0$, $M = 2.0$, $Q_1 = 2.0$, $Du = 0.1$, $\phi_1 = 0.3$, $F = 2.0$, $\gamma = 0.5$, $\phi = 1.0$.

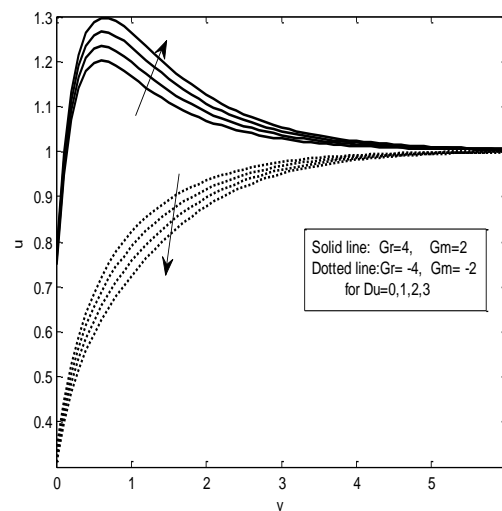


Fig.10: Velocity profiles for various values Du with $Sc = 0.6$, $Q_1 = 2.0$, $Du = 0.5$, $\phi_1 = 0.3$, $F = 2.0$, $\phi = 1.0$, $\gamma = 0.5$, $M = 2.0$.

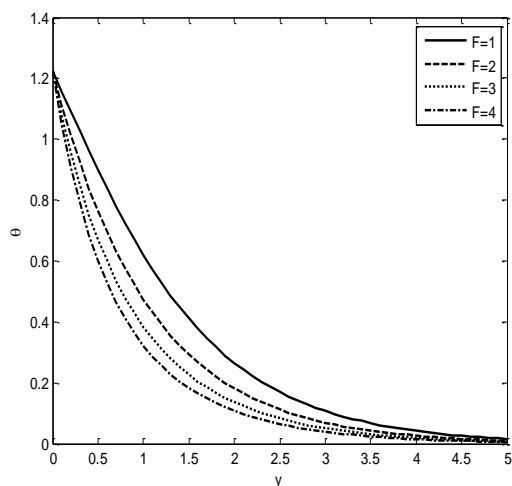


Fig.11: Temperature profiles for various values of F with $Sc = 0.6, Gr = 4.0, Gm = 3.0, M = 2.0, Q_1 = 2.0, Du = 0.5, \phi_1 = 0.3, \phi = 1.0$.

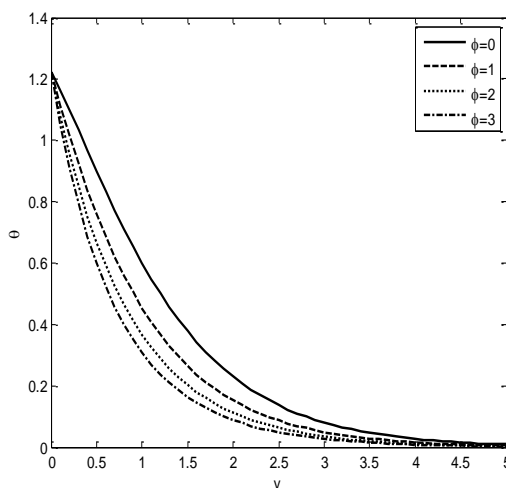


Fig.12: Temperature profiles for various values of ϕ with $Sc = 0.6, Gr = 4.0, Gm = 3.0, M = 2.0, Q_1 = 2.0, Du = 0.5, \phi_1 = 0.3, F = 2.0, \gamma = 1.0$.

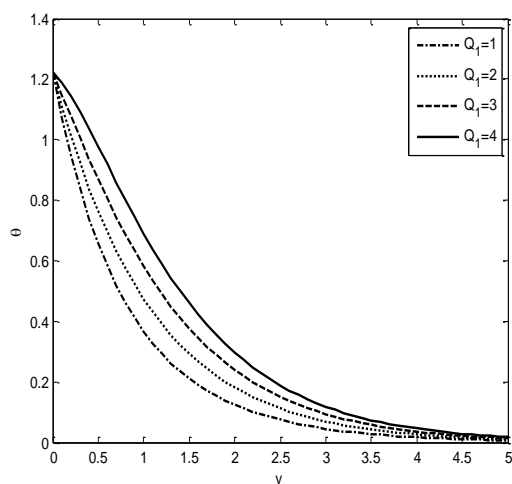


Fig.13: Temperature profiles for various values of Q_1 with $Sc = 0.6, Gr = 4.0, Gm = 3.0, M = 2.0, Du = 0.5, F = 2.0, \phi = 1.0, \gamma = 0.5$.

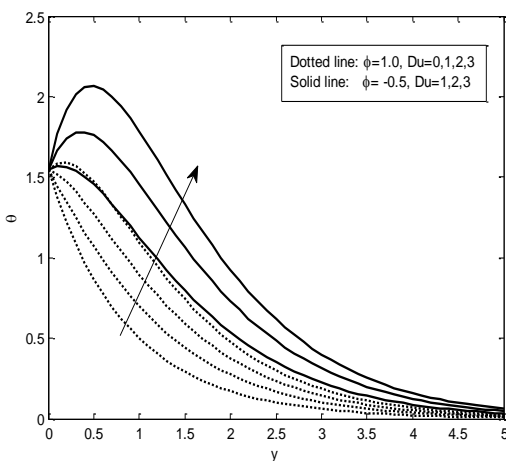


Fig. 14: Temperature profiles for various Du with $Sc = 0.6, Q_1 = 2.0, F = 2.0, \gamma = 0.5$.

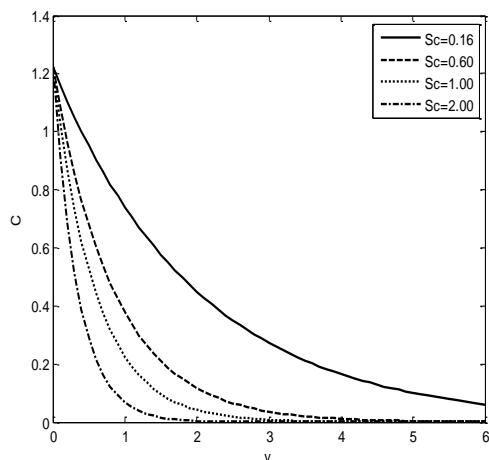


Fig.15: Concentration profiles for various values of Sc with $\gamma = 0.5$.

Table.1: Comparison of present results with those of Kim [23] & Pal and Talukdar [6] with different values of C_{fx} ,

$$Nu_x/Re_x$$

Kim[23] ($G = 2, K = \infty, U_p = 0$)	Pal and Talukdar [6] ($Gr = 2, \phi = 0, \phi_1 = 0, F = 0,$ $Q_1 = 0, Gm = 0$)	Present Results $Gr = 2, \phi = 0, \phi_1 = 0,$ $F = 0, Q_1 = 0, Gm$ $= 0, Du = 0$
$M \quad C_{fx} \quad Nu_x/Re_x$	$M \quad C_{fx} \quad Nu_x/Re_x$	$M \quad C_{fx} \quad Nu_x/Re_x$
0 4.5383 -0.9430	0 4.5383 -0.9430	0 4.5383 -0.9430
2 3.9234 -0.9430	2 3.9234 -0.9430	2 3.9234 -0.9430
5 4.4457 -0.9430	5 4.4457 -0.9430	5 4.4457 -0.9430
10 5.2976 -0.9430	10 5.2976 -0.9430	10 5.2976 -0.9430

Table: 2: Comparison of present results with those of Pal and Talukdar [6] with different values of C_{fx} , Nu_x/Re_x ($F =$

$\gamma \quad C_{fx} \quad Nu_x/Re_x \quad Sh_x/Re_x$ (Pal and Talukdar [6])	$C_{fx} \quad Nu_x/Re_x \quad Sh_x/Re_x$ (Present results)
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$$2, \phi = 1, M = 2, Q_1 = 2, Gr = 4, Gm = 2, Sc = 0.6, \phi_1 = 0.3, Du = 0.0)$$

0.0	0	4.0441	-1.3400	-0.8098	4.0442	-1.3400	-0.8098
0.50		3.7512	-1.4825	-1.1864	3.7513	-1.4825	-1.1864
0.75		3.6744	-1.5226	-1.3178	3.6744	-1.5227	-1.3178
1.00		3.6149	-1.5546	-1.4325	3.6149	-1.5546	-1.4326

Table.3: Effects of ϕ_1 ($F = 2$) and F ($\phi_1 = 0.3$) for fixed values of $Gr = 4$, $Gm = 2$, $Sc = 0.6$, $M = 2$, $\phi = 1$, $Q_1 = 2$, $\gamma = 0.1$, $Du = 0.5$

ϕ_1	C_{fx}	F	C_{fx}	Nu_x / Re_x
0.0	6.2727	1.0	3.6557	-0.8319
0.1	4.9698	2.0	3.5111	-1.3014
0.3	3.5111	3.0	3.4129	-1.6742
0.5	2.7143	4.0	3.3400	-1.9860

The effect of radiation parameter F is shown in fig.1. It is observed in this figure that velocity profiles decrease with an increase in the radiation parameter F . It is clear that, the increase of the radiation parameter leads to decrease the momentum boundary layer thickness and reduces the heat transfer rate in the presence of thermal and solutal buoyancy forces. Fig.2 illustrates the effect of chemical reaction parameter γ . It is clear that at first the velocity increases and later on decreases uniformly with the increasing values of γ . It is interesting to observe that the peak values of the velocity profiles attain near the porous boundary surface. The velocity profiles for different values of magnetic parameter M are depicted in fig.3. From this figure it is clear that as the magnetic field parameter increases, the Lorentz force, which opposes the fluid flow also increases and leads to an enhance deceleration of the flow. This result qualitatively agrees with the expectations since the magnetic field exerts retarding force on the free convection flow. Fig.4 represents the fluid velocity decreases as the heat absorption parameter ϕ increases and hence momentum boundary layer thickness decrease. Figs. 5 and 13 display the effects of the radiation absorption parameter Q_1 on velocity and temperature fields. It is obvious from the figures that an increase in the absorption radiation parameter Q_1 results an increase in the velocity and temperature profiles within the boundary layer as well as an increase in the momentum and thermal thickness. This is because the large values of Q_1

correspond to an increased domination of conduction over absorption radiation, thereby increasing buoyancy force and thickness of the thermal and momentum boundary layer. From fig.5, it is clear that the velocity starts from minimum value of zero at the surface and increases till attains the peak value. For different values of the thermal buoyancy force parameter Gr and solutal buoyancy force parameter Gm are plotted in figs. 6 and 7. As seen from figures that maximum peak value is attained and minimum peak value is observed in the absence of buoyancy force. This is due to fact that buoyancy force enhances fluid velocity and increases the layer thickness with increase in the value of Gr or Gm . Fig.8 illustrates the variation of velocity distribution across the boundary layer for various values of the slip parameter ϕ_1 . It is observed that the velocity increases near the source and reaches the free stream condition. The permeability ϕ_1 is directly proportional to square root of the actual permeability K . Hence, an increase in ϕ_1 will decrease the resistance of the porous medium which will tend to accelerate the flow and increase the velocity.

The influence of Schmidt number Sc on the velocity profiles are shown in fig.9. We observe in fig.9 that at very low values of Schmidt number (e.g., $Sc = 0.16$), there is an increase in the peak velocity near the plate ($y \approx 1$). Whereas for higher values of Schmidt number, the peak shifts closer to the plate. Further it is

observed that the momentum boundary layer decreases with an increase in the value of Sc .

The temperature profiles for different values of the radiation parameter F are shown in fig.11. It is observed that an increase in the radiation parameter F results a decrease of the thermal boundary layer thickness. Further it is observed from that, the temperature is very high at the porous boundary and asymptotically decreases to zero as $y \rightarrow \infty$. Fig.12 depicts the variations in temperature profile against spanwise co-ordinate y for different values of heat absorption parameter ϕ . From this figure, it is clearly understood that heat absorption parameter condenses the thickness of the temperature boundary layer because when heat is absorbed, the buoyancy force decreases the temperature. The effect of Schmidt number Sc on the species concentration profiles is shown in fig.15. It is clear that the concentration decreases exponentially and attains free stream condition. Also it is noticed that the concentration boundary layer thickness decreases with Sc .

The temperature profiles for different values of Dufour number in heat absorption ($\phi > 0$) and heat generation ($\phi < 0$) cases are shown in fig.14. It can be seen that the fluid temperature increases with Dufour number in both the cases. Thermal boundary layer thickness is higher in the case of heat absorption than that in the case of heat generation. Physically, the Dufour term that appears in the temperature equation measures the contribution of concentration gradient to thermal energy flux in the flow domain. It has a vital role in enhancing the flow velocity and the ability to increase the thermal energy in the boundary layer. As a result, the temperature profile at all time stages increases with the increase in Du .

Fig.10 displays the velocity profiles for various values Dufour number in the cases of cooling and heating of the plate. The velocity increases with Dufour number in the case of heating of the plate and opposite trend is observed in the case of cooling of the plate.

From Table.1 and Table.2, it is observed that the skin friction at the plate decreases with increasing chemical reaction parameter γ or Magnetic parameter M . It is also noticed that the Nusselt number and Sherwood number decreases with increase in γ .

The influence of slip parameter ϕ_1 on skin-friction, and radiation parameter F on skin-friction and Nusselt number

are presented in Table. 3. It is observed that the skin friction at the plate decreases with increasing ϕ_1 or F . As the radiation parameter increases the heat is transferred from the plate to the fluid.

V. COMPARISON OF RESULTS

In order to examine the accuracy of the results of the present study, it is considered that the analytical solutions obtained by Kim [23], Pal and Talukdar [6] who computed the numerical results

for skin friction coefficient and Nusselt number. These computed and compared results are presented in Table.1. From this table it is interesting to observe that the present results in the absence of Diffusion-thermo effect and Porous medium are in good agreement with the corresponding results obtained from Pal and Talukdar [6]. It is also observed that the present results are in good agreement with those of Kim [23] when $G = 2, u_p = 0, Du = 0, Gm = 0$. Also from Table. 2, it is clear that the present results for skin-friction, Nusselt number and Sherwood number for different values of chemical reaction in the absence of porous medium and diffusion thermo effect are in good agreement with the corresponding results of Pal and Talukdar [6], which clearly shows the correctness of our present analytical solutions and computed results.

VI. CONCLUSIONS

In this paper we have studied the Dufour effect on an unsteady MHD convective heat and mass transfer flow through a high porous medium over a vertical porous plate. From the present study the following conclusions can be drawn.

1. The diffusion-thermo parameter increases the thermal and momentum boundary layer thickness.
2. The fluid velocity increases with an increasing values of slip parameter ϕ_1 .
3. The skin friction at the plate increases with an increasing values of Du .
4. The rate of heat transfer coefficient at the plate (Nu_x) increase with Dufour number while it is decreases as chemical reaction parameter (γ) or Radiation parameter (F) increases.

Appendix

$$P_1 = \frac{Sc + \sqrt{Sc^2 + 4Sc\gamma}}{2}, m_2 = \frac{Pr + \sqrt{Pr^2 + 4(F + \phi)Pr}}{2}, m_3 = \frac{1 + \sqrt{4(M_1 + n)}}{2},$$

$$m_4 = \frac{Sc + \sqrt{Sc^2 + 4Sc(\gamma + n)}}{2}, m_5 = \frac{Pr + \sqrt{Pr^2 + 4Pr(F + \phi + n)}}{2}, m_6 = \frac{1 + \sqrt{1 + 4M_1}}{2}, A_1 = \frac{(-PrQ_1 - DuP_1^2)}{(P_1^2 - PrP_1 - (F + \phi)Pr)},$$

$$A_2 = \frac{(GrA_1 + Gm)}{(P_1^2 - P_1 - M_1)}, A_3 = \frac{Gr(1 - A_1)}{(m_2^2 - m_2 - M_1)}, A_4 = \frac{Am_6B_4}{(m_6^2 - m_6 - (M_1 + n))}, A_5 = \frac{AScP_1}{(P_1^2 - P_1Sc - Sc(\gamma + n))}$$

$$A_6 = \frac{(-PrQ_1A_5 - DuA_5P_1^2 + PrP_1AA_1)}{(P_1^2 - PrP_1 - Pr(F + \phi + n))}, A_7 = \frac{(PrAm_2 - PrAA_1m_2)}{(m_2^2 - m_2Pr - Pr(F + \phi + n))}, A_8 = \frac{(-PrQ_1 + PrA_5Q_1 - Dum_4^2 + DuA_5m_4^2)}{(m_4^2 - m_4Pr - Pr(F + \phi + n))}$$

$$A_9 = 1 - (A_6 + A_7 + A_8), A_{10} = \frac{AA_3m_2 + GrA_7}{(m_2^2 - m_2 - (M_1 + n))}, B_1 = \frac{AA_2P_1 + GrA_6 + GmA_5}{P_1^2 - P_1 - (M_1 + n)}, B_2 = \frac{GrA_9}{m_5^2 - m_5 - (M_1 + n)}$$

$$B_3 = \frac{GrA_8 + Gm(1 - A_5)}{m_4^2 - m_4 - (M_1 + n)}, B_4 = \frac{(-1 + A_2 + A_3 + A_2\phi_1P_1 + A_3\phi_1m_2)}{(1 + \phi_1m_6)}, B_5 = \frac{-(1 + A_4 - A_{10} - B_1 - B_2 - B_3)}{(1 + \phi_1m_3)} + b_1,$$

$$b_1 = \frac{\phi_1(-A_4m_6 + A_{10}m_2 + B_1P_1 + B_2m_5 + B_3m_4)}{(1 + \phi_1m_3)}$$

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